Chebyshev Based Spectral Representations of Neutron-Star Equations of State

Tianji Zhou, advised by Dr. Lee Lindblom

Department of Physics and Astronomy, Haverford College, and Department of Physics, University of California at San Diego

Introduction

- The equation of state describes the relationship between pressure (P) and energy density (ϵ);
- Given an equation of state $\epsilon = \epsilon(P)$, Oppenheimer and Volkoff solved the Einstein equations to get

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \tag{1}$$

$$\frac{dP}{dr} = -(\epsilon + P)\frac{m + 4\pi r^3 P}{r(r-2m)}; \tag{2}$$

- Neutron star densities surpass lab limits, with no consensus on models, but mass (M) and radius (R) data can help determine their equations of state [1];
- Aim to represent the equations of state numerically with spectral methods to find the best-fit equations of state with M and R data [2];
- Simple power-law bases have been used in previous studies but show slow convergence when adding more spectral parameters for first-order and second-order phase transitions [3];
- Chebyshev polynomials can serve as a faster-converging orthogonal basis, defined by:

$$T_n(x) = \cos(n \arccos(x)) \tag{3}$$

for $n = 0, 1, 2, 3, \ldots$

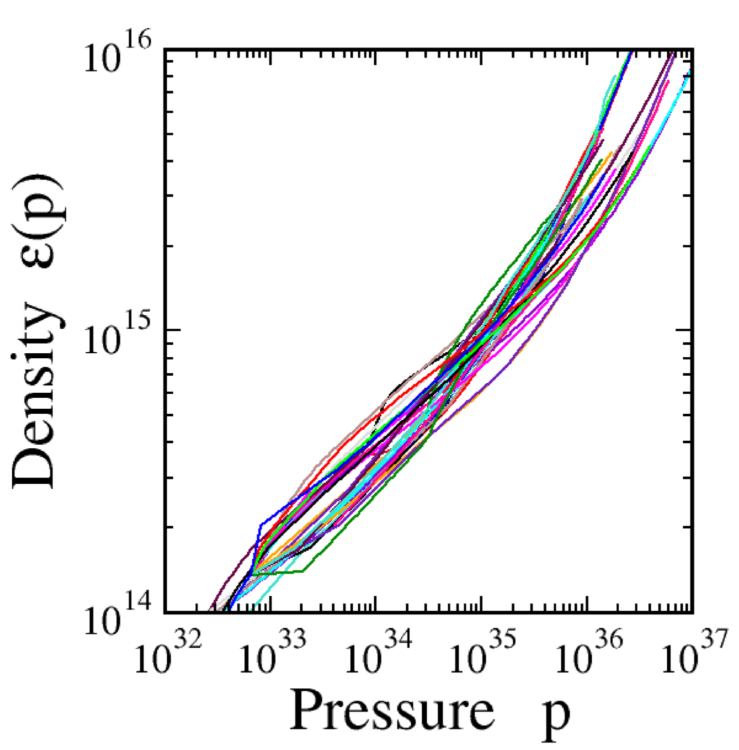

Figure 1: These curves correspond to 34 nuclear-theory models of the neutron stars' equations of state.

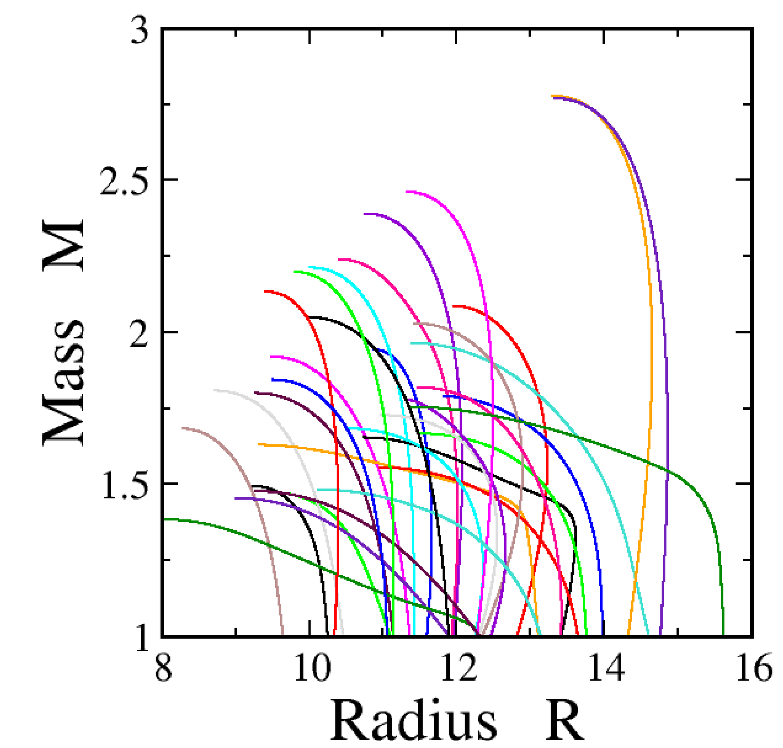

Figure 2: These curves correspond to the equations of state in Figure 1, showing the relationship between mass and radius for different equations of state.

Casual Spectral Representation

- The sound speed of the baryonic fluid is defined by $v^2 = dp/d\epsilon$;
- To ensure the causality of the fluid, we define a velocity function

$$\Upsilon = \frac{c^2 - v^2}{v^2}; \tag{4}$$

- We could either represent using a pressure-based or enthalpy-based approach;
- For pressure-based approach, reduce the sound speed equation to

$$\epsilon(P) = \epsilon_0 + (P - P_0) + \int_{P_0}^{P} \Upsilon(P')\, dP'; \tag{5}$$

- Set Υ to be expressed by spectral method,

$$\Upsilon(P, v_a) = \Upsilon_0 \exp\left\{\sum_{a=0}^{N_{\rm parms}-1} v_a (1+z) T_a(z)\right\}, \tag{6}$$

where v_a is a set of spectral parameters;
- Define

$$z = -1 + 2\log(P/P_0)/\log(P_{max}/P_0). \tag{7}$$

to ensure $-1 \le z \le 1$ for $P_0 \le P \le P_{\rm max}$;
- Enthalpy-based representation is similar to pressure-based representation by re-expressing $P = P(h)$ and $\epsilon = \epsilon(P(h)) = \epsilon(h)$ where h is the enthalpy.

Testing The Representations

- Define the error function $\chi^2 = \frac{1}{N}\sum_{i=0}^{N}[\log(\frac{\epsilon(v_a)}{\epsilon_i})]^2$;
- Using datasets from the first-order and second-order phase transitions to optimize the error function with the best-fitted sets of spectral coefficients;

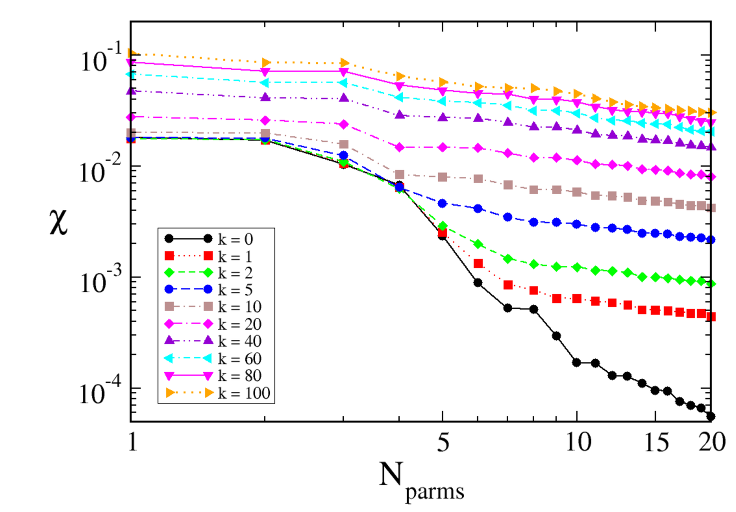

Figure 3: Modeling errors χ using pressure-based Chebyshev polynomial spectral expansions for first-order phase transitions.

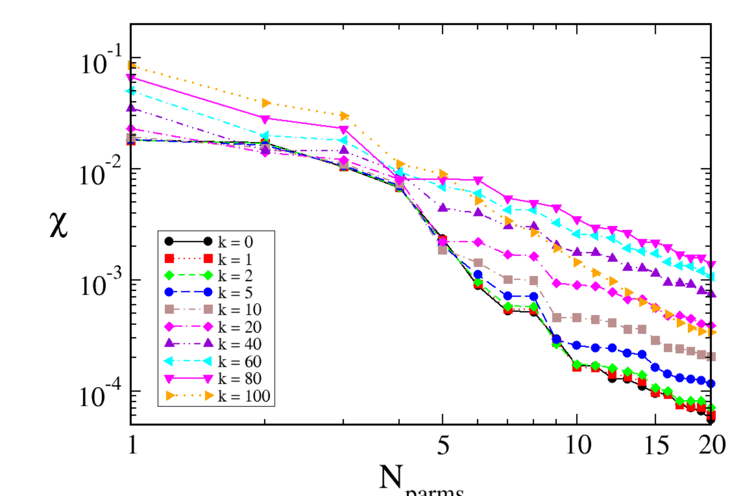

Figure 4: Modeling errors χ using pressure-based Chebyshev polynomial spectral expansions for second-order phase transitions.

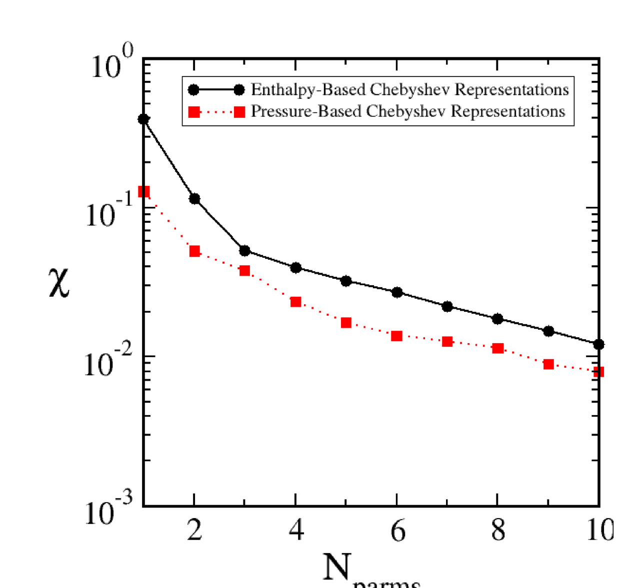

Figure 5: The average modeling error for different nuclear-theory models between Chebyshev enthalpy-based and pressure-based representations.

Conclusion

- When N_{parms} increases, the Chebyshev polynomial representation converges;
- Pressure-based representation does a better job than enthalpy-based representation;
- In the future, we will explore how well the Chebyshev polynomial pressure-based representation models the equations of state by using M and R data with noise.

References

[1] Lee Lindblom.
Determining the Nuclear Equation of State from Neutron-Star Masses and Radii.
ApJ, 398:569, October 1992.

[2] Lee Lindblom.
Spectral representations of neutron-star equations of state.
Phys. Rev. D, 82:103011, Nov 2010.

[3] Lee Lindblom.
Causal representations of neutron-star equations of state.
Phys. Rev. D, 97:123019, Jun 2018.

[4] Lee Lindblom.
Improved spectral representations of neutron-star equations of state.
Phys. Rev. D, 105:063031, Mar 2022.

[5] Lee Lindblom.
Parametric representations of neutron-star equations of state with phase transitions.
Phys. Rev. D, 110:043018, Aug 2024.

Acknowledgements

T.Z. was supported by a Haverford College KINSC Summer Scholars fellowship. The research was conducted at the University of California, San Diego.

HAVERFORD COLLEGE

UC San Diego